# On reducibility of $p$-adic principal series representations of $p-$adic groups

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Abstract

We study the continuous principal series representations of split connected reductive $p$-adic groups over $p$-adic fields. We show that such representations are irreducible when the inducing character lies in a certain cone. This is consistent with a conjecture of Schneider regarding reducibility in the semisimple case.

## 1. Introduction

The theory of $p$-adic Banach space representations of $p$-adic groups was developed by Schneider and Teitelbaum in [ScT02]. These representations play a fundamental role in the $p$-adic Langlands program [BBr10],[Col10]. Important examples of $p$-adic Banach space representations are continuous principal series. In [Sch06], Schneider formulated a conjecture about the irreducibility of principal series representations. In this paper, we confirm the validity of the conjecture for certain characters. These characters form a "cone" in the group of characters, as we will explain below.

Let $\mathbb{Q}_p \subseteq L \subseteq K$ be a sequence of finite extensions. Let $\mathbf{G}$ be a split and connected reductive algebraic $\mathbb{Z}$-group, and $G = \mathbf{G}(L)$. We fix a maximal split torus $\mathbf{T}$ in $\mathbf{G}$ and a minimal parabolic subgroup $\mathbf{P}$ containing $\mathbf{T}$. Set $T = \mathbf{T}(L)$ and $P = \mathbf{P}(L)$.

Let $\chi : T \to K^\times$ be a continuous character. Let

$$\mathrm{Ind}_P^G(\chi^{-1}) = \{f : G \to K \text{ continuous } \mid f(gp) = \chi(p)f(g)\, \forall p \in P, g \in G\},$$

where $G$ acts on the left by $g \cdot f(h) = f(g^{-1}h)$.

Let $X(\mathbf{T})$ be the lattice of rational characters of $\mathbf{T}$. We select a basis $\lambda_1, \dots, \lambda_r$ for $X(\mathbf{T})$ consisting of dominant elements. If $\eta : L^\times \to K^\times$ is a continuous character, then there exists an integer $e(\eta)$ such that $ord_K \circ \eta = e(\eta) \cdot ord_L$ (see Definition 14). Our main result is the following theorem.

Theorem 1. *Let $\chi_1, \dots, \chi_r : L^\times \to K^\times$ be continuous characters such that $e(\chi_i) < 0$ for $1 \leqslant i \leqslant r$. Define $\chi : T \to K^\times$ by $\chi(t) = \prod_{i=1}^r \chi_i(t^{\lambda_i})$. Then $\mathrm{Ind}_P^G \chi^{-1}$ is topologically irreducible (that is, it has no proper nontrivial closed invariant subspaces).*

To explain how this theorem relates to Conjecture 2.5 of [Sch06], assume that $G$ is semisimple and simply connected. Then we can take $\lambda_1, \dots, \lambda_r$ to be the fundamental weights. Let $\delta = \sum_{i=1}^r \lambda_i$. The character $\chi : T \to K^\times$ is called anti-dominant if $\chi\delta \circ \alpha^\vee \neq (\,)^m$ for any integer $m \geqslant 1$ and any positive root $\alpha$. In [Sch06], Schneider conjectures that the $G$-representation

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$\mathrm{Ind}_P^G(\chi^{-1})$ is topologically irreducible if $\chi$ is anti-dominant. The conjecture is known to be true for $G = GL_2(\mathbb{Q}_p)$.

Theorem 1 proves irreducibility for the characters $\chi$ belonging to the cone $\{\chi(t) = \prod_{i=1}^r \chi_i(t^{\lambda_i}) \mid e(\chi_i) < 0 \text{ for } 1 \leqslant i \leqslant r\}$. These characters are anti-dominant. However, the set of anti-dominat elements is much larger than the cone. Still, the value of Theorem 1 is in its generality: $L$ is any finite extension of $\mathbb{Q}_p$ and $G$ is a split connected reductive $L$-group.

The proof of the main theorem relies on the duality theory developed by Schneider and Teitelbaum in [ScT02]. Let $o_L$ denote the ring of integers of $L$. Set $G_0 = \mathbf{G}(o_L)$, $P_0 = \mathbf{P}(o_L)$ and $T_0 = \mathbf{T}(o_L)$. Denote by $\chi_0$ the restriction of $\chi$ to $T_0$. Let $K[[G_0]]$ be the completed group algebra defined in section 6.2. The dual of $\mathrm{Ind}_{P_0}^{G_0}(\chi_0^{-1})$ is $K[[G_0]] \otimes_{K[[P_0]]} K^{(\chi_0)}$. Then the isomorphism $\mathrm{Ind}_P^G(\chi^{-1}) \cong \mathrm{Ind}_{P_0}^{G_0}(\chi_0^{-1})$ induces a $G$-module structure on $M^{(\chi)} = K[[G_0]] \otimes_{K[[P_0]]} K^{(\chi_0)}$. For $\chi$ as in Theorem 1, we prove that $M^{(\chi)}$ is a simple module over the ring $K[G] \otimes_{K[G_0]} K[[G_0]]$ (Theorem 17).

We briefly describe the content of the paper. In section 2, we introduce notation. In section 3 we recall some results that we need from the theory of algebraic representations. These results are used in section 4 to construct a convenient, explicit, model for the space $G/P$. The main technical result of the paper concerns the action of $T$ on this model, and is proved in section 5. In section 6 we recall some facts about principal series representations and their duals, and deduce information about the action of $T$ on these vector spaces from the main technical result concerning its action on $G/P$. Finally, in section 7, we prove the main theorem.

Our group $G$ will be a split connected reductive $L$-group. The group $G$ is determined (up to an $L$-isomorphism) by its root datum ([Spr98], Theorem 16.3.2). Since we also need the corresponding group of $o_L$-points, we use the existence of the split reductive $\mathbb{Z}$-group with the same root datum ([SGA3], XXV.1.2).

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## 2. Notation

Let $L$ be a finite extension of $\mathbb{Q}_p$, $o_L$ its ring of integers, and $\mathfrak{p}_L$ the unique maximal ideal of $o_L$. We denote the discrete valuation of $L$ by $ord_L$. Let $K$ be a finite extension of $L$ and define $o_K, \mathfrak{p}_K$ and $ord_K$ analogously.

We work with algebraic $\mathbb{Z}$-groups as defined in [Jan03]. We denote such an algebraic group by a boldface letter, such as $\mathbf{H}$. Then $H = \mathbf{H}(L)$ is the $L$-points, while $H_0 := \mathbf{H}(o_L)$ is the $o_L$-points. For each integer $n$, there is a canonical projection $H_0 \to \mathbf{H}(o_L/\mathfrak{p}_L^n)$. We denote the kernel by $H_n$.

Let $\mathbf{G}$ be a split and connected reductive algebraic $\mathbb{Z}$-group. We fix a split maximal torus $\mathbf{T}$ and $\mathbf{T}$-stable maximal unipotent subgroup $\mathbf{U}$. We let $\mathbf{P} = \mathbf{TU}$ be the corresponding minimal parabolic subgroup. Also, let $\mathbf{U}^-$ denote the opposite minimal parabolic subgroup.

For each root $\alpha$ of $\mathbf{T}$ in $\mathbf{G}$ we write $\mathbf{U}_\alpha$ for the corresponding root subgroup. We let $W$ denote the Weyl group of $\mathbf{G}$ which we realize as a quotient of the normalizer $N_{\mathbf{G}}(\mathbf{T})$ of $\mathbf{T}$ in $\mathbf{G}$. We fix a set $\dot{W}$ of representatives for $W$ in $G_0$, but do not assume that they form a subgroup. We write $X(\mathbf{T})$ for the lattice of rational characters of $T$. We define a partial order on it by declaring that $\lambda > \mu$ if $\lambda - \mu$ is a sum of positive roots. The group action of $N_{\mathbf{G}}(\mathbf{T})$ on $\mathbf{T}$ by conjugation induce actions of $W$ on $\mathbf{T}$ and $X(\mathbf{T})$. An element of $X(\mathbf{T})$ is said to be dominant if it is maximal in its $W$-orbit, with respect to the above partial ordering. An equivalent condition is that $\lambda$ is dominant if $\langle \lambda, \alpha^\vee \rangle$ is positive for each simple root $\alpha$. This extends the definition of "dominant" to the real vector space $X(\mathbf{T}) \otimes_{\mathbb{Z}} \mathbb{R}$.

LEMMA 2. *The lattice $X(\mathbf{T})$ has a $\mathbb{Z}$-basis which consists of dominant elements.*

*Proof.* Write $\mathcal{D}$ for the set of dominant elements in $X(\mathbf{T}) \otimes_{\mathbb{Z}} \mathbb{R}$. Then

(i) $\mathcal{D}$ is open,

(ii) $a\mathcal{D} = \mathcal{D}$ for any positive real number $a$, and

(iii) $\mathcal{D}$ is convex.

Any subset of $X(\mathbf{T}) \otimes_{\mathbb{Z}} \mathbb{R}$ with the first two properties has the additional property that $\mathcal{D} \cap X(\mathbf{T})$ contains a basis for $X(\mathbf{T}) \otimes_{\mathbb{Z}} \mathbb{R}$. Fix some such basis $\underline{\lambda} := \lambda_1, \dots, \lambda_r$ and let $\mathcal{P}(\underline{\lambda})$ denote the fundamental parallelepiped $\{t_1\lambda_1 + \cdots + t_r\lambda_r : t_1, \dots, t_r \in [0, 1)\}$. If $\mathcal{P}(\underline{\lambda}) \cap X(\mathbf{T}) = \{0\}$, then $\underline{\lambda}$ is a $\mathbb{Z}$=basis for $X(\mathbf{T})$. If not, then we may replace one of the elements of $\underline{\lambda}$ with a nontrivial element of $\mathcal{P}(\underline{\lambda}) \cap X(\mathbf{T})$ to obtain a new basis of $X(\mathbf{T}) \otimes_{\mathbb{Z}} \mathbb{R}$ with a properly smaller fundamental parallelepiped. As $\mathcal{D}$ is convex, the new basis is still contained in $\mathcal{D} \cap X(\mathbf{T})$. Thus if $\underline{\lambda}$ is chosen with $\mathcal{P}(\underline{\lambda})$ minimal, then $\underline{\lambda}$ is a $\mathbb{Z}$-basis for $X(\mathbf{T})$ contained in $\mathcal{D}$. □

## 3. Some results from the theory of algebraic representations

THEOREM 3. *Let $\lambda \in X(\mathbf{T})$ be dominant. Then*

(i) *There is an irreducible finite dimensional algebraic representation $V(\lambda)$ of $\mathbf{G}$ such that $V(\lambda)^{\mathbf{U}}$ is a one-dimensional space on which $\mathbf{T}$ acts by $\lambda$. It is unique up to isomorphism.*

(ii) *If $\alpha$ is a positive root, then $\mathbf{U}_{-\alpha}$ acts trivially on $V(\lambda)^{\mathbf{U}}$ if and only if $\langle \alpha^\vee, \lambda \rangle = 0$.*

(iii) *The representation $V(\lambda)$ has a basis $\mathcal{B}$ consisting of weight vectors such that*

  (a) *the $o_L$-span of $\mathcal{B}$ is preserved by the action of $G_0$*

  (b) *if $v$ is a weight vector on which $\mathbf{T}$ acts by $\lambda$ and $u \in \mathbf{U}_\alpha(o_L)$ then $u.v - v$ is in the span of*

$$\{b \in \mathcal{B} : \mathbf{T} \text{ acts on } b \text{ by } \lambda + n\alpha, \text{ some positive integer } n\}.$$

  (c) *if $v$ is in the $o_L$ span of $\mathcal{B}$ and $u \in \mathbf{U}_\alpha(\mathfrak{p}_L)$ then $u.v - v$ is in the $\mathfrak{p}_L$-span of $\mathcal{B}$.*

*Proof.* (1) follows from [Jan03], proposition 2.4.

Now fix $\lambda$ and for the remainder of the proof denote $V(\lambda)$ more briefly by $V$. Then (2) follows from the representation theory of $\mathfrak{sl}_2$. Indeed, if $v \in V$ then $v$ generates an irreducible representation of the copy of $\mathfrak{sl}_2$ generated by the root subgroups attached to $\pm\alpha$. If $v \in V^{\mathbf{U}}$ then it is also a highest weight vector in this $\mathfrak{sl}_2$-module, and the highest weight is $\langle \lambda, \alpha^\vee \rangle$. The module is one dimensional if and only if its highest weight is 0. Thus, $v$ is annihilated by the Lie algebra of $\mathbf{U}_{-\alpha}$ if and only if $\langle \lambda, \alpha^\vee \rangle = 0$. And this is equivalent to being fixed by $\mathbf{U}_{-\alpha}$ itself.

(3) Let $\mathfrak{g} = \mathrm{Lie}(\mathbf{G})$. Denote by $\mathcal{U}$ the universal enveloping algebra of $\mathfrak{g}$ and $\mathcal{U}_{\mathbb{Z}}$ its Kostant $\mathbb{Z}$-form. Let $V$ be the simple $G$-module of highest weight $\lambda$, with a highest weight vector $v^+$. Define

$$V_{\mathbb{Z}} = \mathcal{U}_{\mathbb{Z}} v^+, \quad V_0 = V_{\mathbb{Z}} \otimes_{\mathbb{Z}} o_L.$$

Then $V_0$ is a $G_0$-invariant lattice in $V$. (Indeed, it's clear that $V_{\mathbb{Z}}$ is fixed by $\mathcal{U}_{\mathbb{Z}}$ and follows that it is fixed by $U_\alpha(\mathbb{Z})$ for each root $\alpha$. The group $T_0$ acts on both $U_{\mathbb{Z}}$ and $v^+$ by elements of $o_L^\times$, and therefore preserves $V_0$. Taken together, these subgroups generate $G_0$.)

The $T_0$-module $V_0$ decomposes into a direct sum of weight spaces

$$V_0 = \bigoplus_{\mu \in X(\mathbf{T})} (V_0)_\mu.$$

We have

$$V_\mu = (V_0)_\mu \otimes_{o_L} L.$$

For each weight space $(V_0)_\mu$, select an $o_L$-basis $\mathcal{B}_\mu$. Let $\mathcal{B} = \bigcup_\mu \mathcal{B}_\mu$. Then (a) is satisfied, and (b) and (c) follow from the expression

$$x_\alpha(a)(m \otimes 1) = \sum_{n \geqslant 0} (X_{\alpha,n} m) \otimes a^n. \tag{1}$$

from [Jan03], Part II, 1.19, eq. (6). □

## 4. A convenient model for $G/P$

Let $\lambda_1, \dots, \lambda_r$ be a basis for $X(\mathbf{T})$ consisting of dominant elements. For each $i$, let $V_i = V(\lambda_i)$ be the unique irreducible representation of $\mathbf{G}$ with highest weight $\lambda_i$, fix a basis $\mathcal{B}_i$ of weight vectors whose $o_L$ span is preserved by $G_0$, and let $v_i$ be the highest weight vector in this basis. Then let $V := \bigoplus_{i=1}^r V_i$. It is equipped with the obvious action of $G$. Let $\mathcal{B} = \mathcal{B}_1 \cup \cdots \cup \mathcal{B}_r$. Let

$$x_0 := (v_i)_{i=1}^r \in V.$$

Lemma 4. *For every $n \in \mathbb{N}$ there exists $k \geqslant n$ satisfying: if $u \in \mathbf{U}_\alpha(L)$, $\alpha$ is negative, and $u.x_0 - x_0$ is in the $\mathfrak{p}_L^k$-span of $\mathcal{B}$, then $u \in \mathbf{U}_\alpha(\mathfrak{p}_L^n)$.*

*Proof.* For each positive root $\alpha$, we select $\lambda_\alpha \in \{\lambda_1, \dots, \lambda_r\}$ such that

$$m_\alpha = \langle \lambda_\alpha, \alpha \rangle \neq 0.$$

Define $m = \max\{m_\alpha \mid \alpha \in \Phi^+\}$. Let $V^\alpha = V(\lambda_\alpha)$. We denote the corresponding highest weight vector by $v_\alpha$.

We have $s_\alpha(\lambda_\alpha) = \lambda_\alpha - m_\alpha \alpha$, so $s_\alpha(V^\alpha_{\lambda_\alpha}) = V^\alpha_{\lambda_\alpha - m_\alpha \alpha}$. It follows

$$\dim V^\alpha_{\lambda_\alpha - m_\alpha\alpha} = \dim V^\alpha_{\lambda_\alpha} = 1.$$

Let $\mathcal{B}^\alpha_{\lambda_\alpha - m_\alpha\alpha} = \{v_{\lambda_\alpha - m_\alpha\alpha}\}$ be the selected $o_L$-basis of $(V_0^\alpha)_{\lambda_\alpha - m_\alpha\alpha}$. Since $X_{-\alpha, m_\alpha} v_\alpha \neq 0$ and $X_{-\alpha, m_\alpha} v_\alpha \in (V_0^\alpha)_{\lambda_\alpha - m_\alpha\alpha}$, we have

$$X_{-\alpha, m_\alpha} v_\alpha = a_\alpha v_{\lambda_\alpha - m_\alpha\alpha}, \quad a_\alpha \in o_L.$$

Define

$$c = \max\{ord_L(a_\alpha) \mid \alpha \in \Phi^+\}.$$

Given $n \in \mathbb{N}$, let $k = mn + c$. Assume $u \in U_{-\alpha}(L)$, $\alpha \in \Phi^+$, and $u.x_0 - x_0$ is in the $\mathfrak{p}_L^k$-span of $\mathcal{B}$. Write $u = x_{-\alpha}(a)$, $a \in L$. Since $u.v_\alpha - v_\alpha$ is in the $\mathfrak{p}_L^k$-span of $\mathcal{B}$, the same holds for its $(V_0^\alpha)_{\lambda_\alpha - m_\alpha\alpha}$-component

$$a^{m_\alpha} X_{-\alpha, m_\alpha} v_\alpha = a^{m_\alpha} a_\alpha v_{\lambda_\alpha - m_\alpha\alpha}.$$

It follows $a^{m_\alpha} a_\alpha \in \mathfrak{p}_L^k$. Then

$$ord_L(a^{m_\alpha}) \geqslant k - ord_L(a_\alpha) \geqslant mn,$$

so $ord_L(a) \geqslant mn/m_\alpha \geqslant n$. This proves $u \in U_{-\alpha}(\mathfrak{p}_L^n)$. □

DEFINITION 5. *Given $n \in \mathbb{N}$, we define $k(n)$ to be the least integer $\geqslant n$ satisfying the condition of Lemma 4*

We identify $X(\mathbf{P})$ with $X(\mathbf{T})$ via the projection $\mathbf{P} \to \mathbf{T}$. We use exponential notation for rational characters. Then $p \cdot x_0 = (p^{\lambda_i} \cdot v_i)_{i=1}^r$. Let

$$X = \mathbf{G}x_0 \subset V \quad \text{and} \quad X_0 = G_0 x_0.$$

We have the obvious action of $\mathbf{G}$ on $X$. We also have an obvious action of $GL_1^r$ on $V$ by scaling in each factor. As $GL_1^r$ is abelian, this can be viewed as a left or right action and it is convenient to view it as a right action. Explicitly,

$$(x_1, \dots, x_r) \cdot (a_1, \dots, a_r) = (a_1 x_1, \dots, a_r x_r), \qquad x_i \in V_i, a_i \in GL_1, i = 1, \dots, r.$$

We have a homomorphism $\mathbf{P} \to GL_1^r$ given by $p \mapsto (p^{\lambda_i})_{i=1}^r$. Hence we may pull our right action of $GL_1^r$ back to a right action of $\mathbf{P}$.

Write $[V_i]$ for $(V_i \smallsetminus \{0\})/GL_1$ and $[V]$ for $\oplus_{i=1}^r [V_i]$. For $x \in V_i$ (resp. $V$) write $x$ for the image in $[V_i]$ (resp. $[V]$). If $\lambda \in X(\mathbf{T})$, is dominant, let $\mathbf{P}_\lambda$ be the standard parabolic subgroup such that a simple root $\alpha$ of $\mathbf{G}$ is a simple root of the Levi factor if $\langle \lambda, \alpha^\vee \rangle = 0$ and a root of the unipotent radical if and $\langle \lambda, \alpha^\vee \rangle > 0$.

PROPOSITION 6. *We have*

(i) *The stabilizer of $[v_i] \in [V_i]$ is $\mathbf{P}_{\lambda_i}$.*

(ii) *The stabilizer of $[x_0] \in [V]$ is $\mathbf{P}$.*

(iii) *The stabilizer of $v_i \in V_i$ is the kernel of a rational character $\widetilde{\lambda}_i$ whose restriction to $\mathbf{P}_{\lambda_i}$ is $\lambda_i$.*

(iv) *The stabilizer of $x_0 \in V$ is $\mathbf{U}$.*

*Proof.* It's clear from the definitions that $\mathbf{P}$ stabilizes $[v_i]$ for all $i$ and hence also $[x_0]$. By Corollary 21.3.B from [Hum75], the stabilizer of $[v_i]$ is a standard parabolic subgroup for each $i$. It then follows from theorem 3, part (2) that the stabilizer of $[v_i]$ is $\mathbf{P}_{\lambda_i}$ for each $i$. Since $\mathbf{P}_{\lambda_i}$ stabilizes $[v_i]$, it must act on $v_i$ by a rational character. We denote this rational character $\widetilde{\lambda}_i$ and it is evident that the restriction to $\mathbf{T}$ is $\lambda_i$.

The stabilizer of $[x_0]$ is $\bigcap_{i=1}^r \mathbf{P}_{\lambda_i}$ which contains $\mathbf{P}$. On the other hand, since $\lambda_1, \dots, \lambda_r$ span $X(\mathbf{T})$, it follows that for any $0 \neq \varphi \in X^\vee(\mathbf{T})$, there exists $i$ with $\langle \varphi, \lambda_r \rangle \neq 0$. Applying this to the coroots, we deduce that $\bigcap_{i=1}^r \mathbf{P}_{\lambda_i}$ does not contain $\mathbf{U}_\alpha$ for any negative root $\alpha$. Thus $\bigcap_{i=1}^r \mathbf{P}_{\lambda_i} = \mathbf{P}$. Similarly, if $t \in \mathbf{T}$ stabilizes $x_0$, then it is in the kernel of $\lambda_i$ for all $i$, and hence is trivial. $\square$

### 4.1 Coset representatives for $G/P$

We know that $G_0 P = G$. Write $l$ for $o_L/\mathfrak{p}_L$ and $\bar{H}$ for $H(l)$ (identified with the image of $H_0$ in $G(l)$) for any algebraic subgroup of $G$. Write $B$ for the preimage of $\bar{P}$ in $G_0$. So $B = G_1 P_0$. Also, for each $w \in W$, let $\mathbf{U}_w = \mathbf{U} \cap w\mathbf{U}^- w^{-1}$. We have

$$\bar{G} = \coprod_w \bar{U}_w w \bar{P}.$$

Pulling back, we have

$$G_0 = \coprod_w (U_0 \cap wU^- w^{-1}) w G_1 P_0 = \coprod_w U_{w,0} w U_1^- P_0 = \coprod_w w U^-_{w,\frac{1}{2}} P_0,$$

where $U^-_{w,\frac{1}{2}} = w^{-1}U_{w,0}wU_1^-$. Note that $U^-_{w,\frac{1}{2}}$ is a subgroup of $U^-_{w,0}$. Indeed $w^{-1}U_w w = U^- \cap w^{-1}Uw$ is a subgroup of $U^-$, and $U^-_{w,\frac{1}{2}}$ is the preimage of $w^{-1}\bar{U}_w w$ in $U_0^-$. It may be understood explicitly as follows. Fix an order on the negative roots. Then multiplication gives a bijection $\prod_{\alpha<0} U_\alpha \to U^-$. (This is, not, in general a group homomorphism.) Then $U^-_{w,\frac{1}{2}}$ corresponds to $\prod_{\alpha<0, w\alpha>0} U_{\alpha,0} \times \prod_{\alpha<0,w\alpha<0} U_{\alpha,1}$.

This can also be written as

$$\coprod_w (U_1^- \cap wU^-w^{-1})(U_0 \cap wU^-w^{-1})wP_0.$$

Thus, if we fix a representative $\dot{w}$ in $N_G(T)$ for each $w \in W$, then $\coprod_w \dot{w}U^-_{w,\frac{1}{2}}$ or $\coprod_w (U_1^- \cap wU^-w^{-1})(U_0 \cap wU^-w^{-1})\dot{w}$ maps injectively into $X$ and onto $[X]$.

Let

$$X_0^1 = \coprod_w \dot{w}U^-_{w,\frac{1}{2}} x_0.$$

Since $\coprod_w \dot{w}U^-_{w,\frac{1}{2}}$ is a set of coset representatives for $G/P$, and since $P$ acts on $x_0$ through the map to $GL_1^r$, it follows that

$$\forall x \in X \exists x_0^1 \in X_0^1, a \in GL_1^r \text{ such that } x = x_0^1 \cdot a.$$

Since the stabilizer of $x_0$ is $U$, one can say that $x_0^1$ and $a$ are unique. Since the map $T \to GL_1^r$ is a bijection, we can also say

$$\forall x \in X \exists ! x_0^1 \in X_0^1, t \in T \text{ such that } x = x_0^1 \cdot t.$$

The next lemma permits us to compute $x_0^1$ and $t$ explicitly, given $x$. Moreover, the set $X_0^1$ is given as a disjoint union of components indexed by the elements of $w$, and the next lemma also provides a means of determining which component $x_0^1$ is in.

Lemma 7. *Let $x = (x_1, \dots, x_r)$ be an element of $X_0^1$. For $1 \leqslant i \leqslant r$ let $\mathcal{B}_i = (b_{i,1}, \dots, b_{i,\dim V_i})$ be a basis for $V_i$ as in part(3) of theorem 3. By hypothesis, then, $t$ acts on $b_{i,j}$ by some weight $\lambda_{i,j} \in X(\mathbf{T})$. Write*

$$x_i = \sum_{j=1}^{\dim V_i} c_{ij} b_{i,j}.$$

*Then*

(i) $c_{ij} \in o_L \forall i, j$.
(ii) *For each $i$, there exists $j$ with $c_{i,j} \in o_L^\times$.*
(iii) *For each $i$, $\{\lambda : \exists j \text{ with } \lambda_{i,j} = \lambda \text{ and } c_{i,j} \in o_L^\times\}$ has a unique minimal element. It is $w \cdot \lambda_i$ where $x \in \dot{w}U^-_{w,\frac{1}{2}} \cdot x_0$.*

*Proof.* We know that $x = \dot{w}u.x_0$ for some unique $w$ in the Weyl group and $u \in U^-_{w,\frac{1}{2}}$. Part (1) follows from the fact that both $u$ and $\dot{w}$ are in $G_0$ and the fact that the $o_L$-span of $\mathcal{B}$ is $G_0$-stable.

To prove parts (2) and (3), we use the fact that $\dot{w}U^-_{w,\frac{1}{2}} = (U_1^- \cap wU^-w^{-1})(U_0^+ \cap wU^-w^{-1})\dot{w}$, to write $x = u_1u_0\dot{w}.x_0$, where $u_0 \in U_0^+ \cap wU^-w^{-1}$ and $u_1 \in U_1^- \cap wU^-w^{-1}$.

Let $v_i' = \dot{w}v_i$. Then $t$ acts on $v_i'$ by the $w \cdot \varpi_i$. The space of highest weight vectors is one dimensional, so the space of weight vectors in $V_i$ attached to the weight $w \cdot \lambda_i$ is one-dimensional as well. Thus, there exists $j_0$ such that $v_i' = cb_{i,j_0}$. Further, $c$ is a unit, because $(\dot{w^{-1}})\dot{w} \in T_0$.

Next, write $u_0 \cdot v_i' = \sum_{j=0}^{\dim V_i} d_{i,j} b_{i,j}$, where $d_{i,j} \in o_L$. Then it follows from Theorem 1, part 3(b) that $d_{i,j} = 0$ unless $\lambda_{i,j} - w \cdot \lambda_i$ is zero or a sum of positive roots, and $d_{i,j_0} = c$.

It then follows from Theorem 1, part 3(c) that $c_{i,j} \equiv d_{i,j}$ for all $i, j$. In particular, $c_{i,j_0}$ is a unit, and if $j \neq j_0$ and $c_{i,j}$ is a unit, then $d_{i,j}$ is a unit so that $\lambda_{i,j} - w \cdot \lambda_i$ is a sum of positive roots. $\square$

## 5. Key Technical Result

Lemma 8. *Fix a positive integer $n$. Take $x \in X_0^1$ and $t \in T$. Assume that $ord_L(t^\alpha) \geqslant k(n)$ for each simple root $\alpha$. Let $t' \in T$ and $x' \in X_0^1$ be the unique elements satisfying $t \cdot x = x' \cdot t'$. Then*

$$ord_L((t')^{\lambda_i}) \leqslant ord_L(t^{\lambda_i}), \qquad (1 \leqslant i \leqslant r)$$

*and equality holds in all places if and only if $x \in U_n^- x_0$.*

*Proof.* Set $t \cdot x =: y =: (y_1, \dots, y_r)$ with

$$y_i = \sum_{j=1}^{\dim V_i} y_{i,j} b_{i,j} = \sum_{j=1}^{\dim V_i} t^{\lambda_{i,j}} x_{i,j} b_{i,j}$$

Then

$$ord_L((t')^{\lambda_i}) = \min_j ord_L(y_{i,j}) = \min_j (ord_L(t^{\lambda_{i,j}} x_{i,j})).$$

Now, if $\lambda$ is a weight of $T$ in $V_i$, then $\lambda_i - \lambda$ is a sum of positive roots. Let us assume that the numbering is such that $b_{i,1}$ is the highest weight vector. Then $ord_L(t^{\lambda_i - \lambda_{i,j}}) \geqslant k(n)$ for all $j > 1$ and all $i$.

Now, we know that for each $i$ there exists $j_0$ such that $x_{ij_0} \in o_L^\times$. Hence

$$\min_{1 \leqslant j \leqslant \dim V_i} ord_L(y_{i,j}) \leqslant ord_L(t^{\lambda_{i,j_0}}) \leqslant ord_L(t^{\lambda_i})$$

with equality if and only if $ord_L(x_{i,j}) \geqslant ord_L(t^{\lambda_i - \lambda_{i,j}}) \geqslant k(n)$ for all $j > 1$. If this is the case, then $x = u x_0$ for some $u \in U^-_{e,\frac{1}{2}} = U_1^-$. (Here $e$ is the identity in the Weyl group.) In fact, $u \in U_n^-$ by the definition of $k(n)$ $\square$

Corollary 9. *Fix a positive integer $n$. Take $g \in \coprod_w w U^-_{w,\frac{1}{2}}$ and $t \in T$. Assume that $ord_L(t^\alpha) \geqslant k(n)$ for each simple root $\alpha$. Let $t' \in T$ and $g' \in \coprod_w w U^-_{w,\frac{1}{2}}$ be the unique elements satisfying $t \cdot g \in g' \cdot t' U$. Then*

$$ord_L((t')^{\lambda_i}) \leqslant ord_L(t^{\lambda_i}), \qquad (1 \leqslant i \leqslant r)$$

*and equality holds in all places if and only if $g \in U_n^-$.*

*Proof.* Apply the previous to $x = g.x_0$. Then $g'$ is the unique element of $\coprod_w w U^-_{w,\frac{1}{2}}$ such that $x' = g'.x_0$. $\square$

Corollary 10. *Fix a positive integer $n$. Take $g \in G_0$ and $t \in T$. Assume that $ord_L(t^\alpha) \geqslant k(n)$ for each simple root $\alpha$. If $tg \in G_0 t' U$, then*

$$ord_L((t')^{\lambda_i}) \leqslant ord_L(t^{\lambda_i}), \qquad (1 \leqslant i \leqslant r)$$

*and equality holds in all places if and only if $g \in U_n^- P_0$.*

*Proof.* Write $g = g_1 p_0$ with $p_0 \in P_0$ and $g_1 \in \coprod_w wU^-_{w,\frac{1}{2}}$. Use the previous corollary to write $tg_1 = g't''u$ where $g' \in \coprod_w wU^-_{w,\frac{1}{2}}$, $t'' \in T$, and $u \in U$, and get $tg = g't''up_0$. If $tg = g_0 t'U$, then $g_0^{-1}g' \in t'Up_0^{-1}u^{-1}t''^{-1}$. It follows that $t't''^{-1} \in T(o_L)$. □

## 6. Induced representations and their duals

With the required technical result in hand, we are ready to proceed to the main theorem, which applies these technical results to the problem of reducibility of principal series representations of $p$-adic groups over $p$-adic fields. Recall that we have fixed a finite extension $K$ of $L$.

### 6.1 Principal series representations

Let $\chi : T \to K^\times$ be a continuous homomorphism, and let $\chi_0$ denote the restriction of $\chi$ to $T_0$. Also, let

$$I = \mathrm{Ind}_P^G(\chi^{-1}) = \{f \in C(G, K) \mid f(gp) = \chi(p)f(g)\, \forall p \in P, g \in G\},$$

be the corresponding principal series representation. Restriction gives an isomorphism

$$I \cong \mathrm{Ind}_{P_0}^{G_0}(\chi_0^{-1}).$$

### 6.2 Completed group algebras

If $\bar{H}$ is a finite group, we have the usual group algebra $o_K[\bar{H}]$, and the augmentation homomorphism $\mathrm{aug} : o_K[\bar{H}] \to o_K$ given by

$$\mathrm{aug} \sum_{\bar{h}\in\bar{H}} a_{\bar{h}}\bar{h} := \sum_{\bar{h}\in\bar{H}} a_{\bar{h}}.$$

If $H$ is a compact $p$-adic Lie group (such as $G_i$, where $i \geqslant 0$) we have the projective limit $o_K[[H]] := \mathrm{proj\,lim}_{H'}\, o_K[H/H']$ taken over compact open normal subgroups $H'$ of $H$. The augmentation homomorphism extends canonically to $o_K[[H]]$. If $H = G_i$ for some $i$, the projective limit may be taken over the groups $G_j, j \geqslant i$.

The ring $o_K[H/H']$ is canonically identified with the space of $o_K$-linear maps $C(H/H', o_K) \cong C(H, o_K)^{H'} \to o_K$. This induces canonical isomorphisms of $o_K[[H]]$ with the space of $o_K$-linear maps $C(H, o_K) \to o_K$, and of $K[[H]] := K \otimes_{o_K} o_K[[H]]$ with the space of all distributions on $H$ (i.e., $K$-linear maps $C(H, K) \to K$). A distribution $\nu \in K[[H]]$ may be written as $\nu_0 \otimes 1$ with $\nu_0 \in o_K[[H]]$ if and only if it maps $C(H, o_K)$ into $o_K$.

Lemma 11.

$$1 + \varpi_K o_K[[G_0]] \subset o_K[[G_0]]^\times$$

*Proof.* Indeed, if $\mu \in o_K[[G_0]]$, then

$$\sum_{j=0}^{\infty} \varpi_K^j \mu^j$$

converges to an element of $o_K[[G_0]]$ and that multiplying by $(1 - \varpi_K\mu)$ gives 1. □

### 6.3 The dual of a principal series representation

The dual of $Ind_{P_0}^{G_0}(\chi_0^{-1})$ is $K[[G_0]] \otimes_{K[[P_0]]} K^{(\chi_0)}$. Here $K[[H_0]] = K \otimes_{o_K} o_K[[H_0]]$, where $o_K[[H_0]]$ is the projective limit of the group rings $o_K[H_0/H']$ as $H'$ varies over compact open subgroups of $H_0$. To define the projective limit, it suffices to let $H'$ vary over the subgroups $H_n, n \geqslant 0$. The isomorphism $I \cong \mathrm{Ind}_{P_0}^{G_0}(\chi_0^{-1})$. induces a $G$-module structure on $K[[G_0]] \otimes_{K[[P_0]]} K^{(\chi_0)}$ which depends on $\chi$. We denote this $G$-module $M^{(\chi)}$.

LEMMA 12. (i) *The image of $o_K[[G_0]]$ in $M^{(\chi_0)}$ is the set of elements which map $o_K$-valued elements of $Ind_{P_0}^{G_0}\chi_0^{-1}$ into $o_K$.*

(ii) *Likewise, for each integer $r$, the image of $\varpi_K^r \cdot o_K[[G_0]]$ is the set of elements which map $o_K$-valued elements of $Ind_{P_0}^{G_0}\chi_0^{-1}$ into $\mathfrak{p}_K^r$.*

*Proof.* The second part follows easily from the first. It follows easily from the definitions that $\mu \in o_K[[G_0]]$ maps $o_K$-valued continuous functions into $o_K$. Now take $\nu \in K[[G_0]]$ which maps $\mathrm{Ind}_{P_0}^{G_0}\chi_0^{-1}$ into $o_K$. We construct $\mu \in o_K[[G_0]]$ such that $\mu$ and $\nu$ have the same image in the quotient $M^{(\chi)}$. Recall that

$$G_0 = \coprod_w wU^-_{w,\frac{1}{2}}P_0.$$

Hence $C(G_0, o_K) = \oplus_w C(wU^-_{w,\frac{1}{2}}P_0, o_K)$. We write the corresponding decomposition for $Ind_{P_0}^{G_0}(\chi_0^{-1})$. The component corresponding to $w \in W$ is the elements of the induced space supported on the compact open set $wU^-_{w,\frac{1}{2}}P_0$. A vector space isomorphism to $C(U^-_{w,\frac{1}{2}}, o_K)$ is given by $h \mapsto f_h$, where

$$f_h(g) = \begin{cases} h(u)\chi_0(p), & g = wup,\ u \in U^-_{w,\frac{1}{2}},\ p \in P_0 \\ 0, & g \notin wU^-_{w,\frac{1}{2}}P_0. \end{cases}$$

Suppose now that we have $\nu \in M^{(\chi)}$ which maps $Ind_{P_0}^{G_0}(\chi_0^{-1}) \cap C(G_0, o_K)$ into $o_K$. Write $\nu = \oplus_w \nu_w$ with $\nu_w$ supported on $wU^-_{w,\frac{1}{2}}P_0$. Each $\nu_w$ determines a distribution on $C(U^-_{w,\frac{1}{2}}, K)$, mapping $o_K$-valued functions into $o_K$. In other words, an element of $\eta_w \in o_K[[U^-_{w,\frac{1}{2}}]]$. This permits us to construct $\eta_{w,n} \in o_K[U^-_{w,\frac{1}{2}}/U^-_n]$. If

$$\eta_{w,n} = \sum_{\bar{u} \in U^-_{w,\frac{1}{2}}/U^-_n} c_{w,n,\bar{u}}\bar{u},$$

then define

$$\mu_n = \sum_w \sum_{\bar{u} \in U^-_{w,\frac{1}{2}}/U^-_n} c_{w,n,\bar{u}} w\bar{u}.$$

Let $\mu = (\mu_n)_{n=0}^{\infty}$. Then $\mu \in o_K[[G_0]]$ and its image in $M^{(\chi)}$ is $\nu$. □

## 7. Main Theorem

We begin with an important result from [Sch11].

PROPOSITION 13. *Assume $n \geqslant 1$.*

(i) *$o_K[[G_n]]$ is a local ring with residue field $o_K/\mathfrak{p}_K$.*

(ii) *The maximal ideal in $o_K[[G_n]]$ is*

$$\{\mu \in o_K[[G_n]] : \mathrm{aug}(\mu) \in \mathfrak{p}_K\}.$$

Next we need to define an important invariant.

Definition 14. *Let $\eta : L^\times \to K^\times$ be a character (continuous homomorphism). As $\eta$ must map $o_L^\times$ into $o_K^\times$, it induces a map $L^\times/o_L^\times \to K^\times/o_K^\times$. That is, for $a \in L^\times$, $ord_K(\eta(a))$ depends only on $ord_L(a)$. Let $e(\eta)$ denote the integer such that $ord_K \circ \eta = e(\eta) \cdot ord_L$.*

Proposition 15. *Fix a positive integer $n$. Take $t \in T$. Assume that $ord_L(t^\alpha) \geqslant k(n)$ for each simple root $\alpha$. Take $\chi_1, \dots, \chi_r : L^\times \to K^\times$ and assume that $e(\chi_i) < 0$ for $1 \leqslant i \leqslant r$. Define $\chi : T \to K^\times$ by $\chi(t) = \prod_{i=1}^r \chi_i(t^{\lambda_i})$ where $\lambda_1, \dots \lambda_r$ are dominant and form a basis for $X(\mathbf{T})$. Take $f \in Ind_P^G \chi^{-1}$ which maps $G_0$ into $o_K$. Write $L^\iota$ for left-inverse translation. Then $L^\iota(t^{-1}).f$ maps $G_0 \smallsetminus G_n P_0$ into $\chi(t)\mathfrak{p}_K$.*

*Proof.* Take $g \in G_0 \smallsetminus G_n P_0$ $[L^\iota(t^{-1}).f](g) = f(tg)$. Write $tg = g't'u$ where $g' \in G_0, t' \in T$, and $u \in U$. Then $f(tg) = f(g')\chi(t')$. As $f(g') \in o_K$ we just need to show that $\chi(t') \in \chi(t)\mathfrak{p}_K$. Equivalently, we need to show that $ord_K(\chi(t't^{-1})) > 0$. But

$$ord_K(\chi(t't^{-1})) = \sum_{i=1}^r e(\chi_i) \left( ord_L((t')^{\lambda_i}) - ord_L((t)^{\lambda_i}) \right).$$

Corollary 10 shows that each of the integers $ord_L((t')^{\lambda_i}) - ord_L((t)^{\lambda_i})$ is nonpositive and at least one is nonzero. Hence if $e(\chi_i)$ is strictly negative for each $i$, it follows that the sum will be strictly positive. □

Corollary 16. *Fix a positive integer $n$. Take $t \in T$ such that $ord_L(t^\alpha) \geqslant k(n)$ for each simple root $\alpha$, and $\chi_1, \dots, \chi_r : L^\times \to K^\times$, such that $e(\chi_i) < 0$ for $1 \leqslant i \leqslant r$. Define $\chi : T \to K^\times$ by $\chi(t) = \prod_{i=1}^r \chi_i(t^{\lambda_i})$ where $\lambda_1, \dots \lambda_r$ are dominant and form a basis for $X(\mathbf{T})$. Let $\nu \in M^{(\chi)}$ lie in the image of $o_K[[G_0]] < K[[G_0]]$ and vanish on $G_n P$. Then $t \cdot \nu$ lies in the image of $\chi(t)\varpi_K \cdot o_K[[G_0]]$.*

*Proof.* Take $f \in Ind_P^G \chi^{-1}$ which maps $G_0$ into $o_K$. Then

$$\langle t.\nu, f \rangle = \langle \nu, L^\iota(t^{-1}).f \rangle = \langle \nu, \pi_2(L^\iota(t^{-1}).f) \rangle,$$

where $\pi_2$ is projection onto the second factor in the decomposition $C(G, K) = C(G_n P, K) \oplus C(G \smallsetminus G_n P, K)$. By the previous proposition $\pi_2(L^\iota(t^{-1}).f$ maps $G_0$ into $\chi(t)\mathfrak{p}_K$. It then follows that $t.\nu$ maps $f$ into $\chi(t)\mathfrak{p}_K$. □

Theorem 17. *Define $\chi : T \to K^\times$ by $\chi(t) = \prod_{i=1}^r \chi_i(t^{\lambda_i})$ where $\lambda_1, \dots \lambda_r$ are dominant and form a basis for $X(\mathbf{T})$. Assume that $e(\chi_i) < 0$ for $1 \leqslant i \leqslant r$. Then $M^{(\chi)}$ is a simple module over the ring $K[G] \otimes_{K[G_0]} K[[G_0]]$.*

*Proof.* Choose a nontrivial element of $M^{(\chi)}$, and construct a representative $\eta = \sum_{w \in W} w\eta_w$ for it as in the proof of Lemma 11. Here $\eta_w \in K[[U^-_{w,\frac{1}{2}}]]$ for each $w \in W$. By scaling, we may assume that $\eta_w = (\eta_{w,n})_{n=0}^\infty \in o_K[[U^-_{w,\frac{1}{2}}]]$ for each $w$, and that there exists $n \geqslant 1, w_0 \in W$ and $\bar{u}_0 \in U^-_{w,\frac{1}{2}}/U^-_n$ such that the coefficient of $\bar{u}_0$ in $\eta_{w,n}$ is a unit. Choose $u_0 \in U^-_{w,\frac{1}{2}}$ which projects to $\bar{u}_0$, and let $\mu = u_0^{-1} w_0^{-1} \eta$.

Now, observe that the partition of $G_0$ as $G_n \cup (G_0 \smallsetminus G_n)$ gives rise to direct sum decompositions of $C(G_0, K), o_K[[G_0]]$ and $K[[G_0]]$. Moreover $\{\lambda \in K[[G_0]] : \mathrm{supp}(\lambda) \subset G_n\}$ is canonically identified with $K[[G_n]]$. Consider the projection of $\mu$ onto $o_K[[G_n]]$. Its image under the augmentation map is precisely the coefficient of the identity coset in $\mu_n$, i.e., the coefficient of $w_0\bar{u}_0$ in

$\eta_n$. By hypothesis, this is a unit. It follows that the projection of $\mu$ onto $o_K[[G_n]]$ is an invertible element of $o_K[[G_n]]$. Multiplying by its inverse, we obtain an element of the form $1+\nu$ where the support of $\nu$ is disjoint from $G_n$.

Lemma 18. *The support of $\nu$ is, in fact, disjoint from $G_nP_0$.*

*Proof of Lemma.* It suffices to prove that $\mathrm{supp}(\mu) \cap G_nP_0$ is contained in $G_n$. As $\mathrm{supp}(\mu) \subset u_0^{-1}w_0^{-1}\coprod_{w\in W} wU^-_{w,\frac{1}{2}}$, it suffices to show that $u_0^{-1}w_0^{-1}wU^-_{w,\frac{1}{2}} \cap G_nP_0$ is trivial when $w \neq w_0$ and contained in $G_n$ when $w = w_0$.

Consider the projection from $G_0$ to $\bar{G}$ (the points of $\bar{G}$ over the finite field $l = o_L/\mathfrak{p}_L$). The sets $wU^-_{w,\frac{1}{2}}$ for $w \in W$ all project to distinct Bruhat cells. Hence $w_0u_0G_1P_0 \cap wU^-_{w,\frac{1}{2}} \neq \emptyset \implies w = w_0$. But $w = w_0 \implies u_0^{-1}w_0^{-1}wU^-_{w,\frac{1}{2}} \subset U_0^-$, and the only element of $G_0/G_n$ which is in the image of both $P_0$ and $U_0^-$ is the identity. □

Now choose $t$ so that $ord_L(t^\alpha) > k(n)$ for each simple root $\alpha$. Then it follows directly from the definitions that the images of $t\cdot 1$ and $\chi(t)\cdot 1$, in $M^{(\chi)}$ are the same. Futher, by corollary 16, the image of $t\cdot\nu$ in $M^{(\chi)}$ is contained in the image of $\chi(t)\cdot\varpi_K\cdot o_K[[G_0]]$. It follows that, the submodule of $M^{(\chi)}$ generated by $[\mu]$ contains $[1+\chi(t)^{-1}t\cdot\nu]$. But lemma 11 implies that $1+\chi(t)^{-1}t\cdot\nu$ is a unit. Hence the submodule generated by $[\mu]$ is all of $M^{(\chi)}$. □

Theorem 19. *Define $\chi : T \to K^\times$ by $\chi(t) = \prod_{i=1}^r \chi_i(t^{\lambda_i})$ where $\lambda_1, \dots \lambda_r$ are dominant and form a basis for $X(\mathbf{T})$. Assume that $e(\chi_i) < 0$ for $1 \leqslant i \leqslant r$. Then $\mathrm{Ind}_P^G \chi^{-1}$ is topologically irreducible (that is, it has no proper nontrivial closed invariant subspaces).*

*Proof.* Follows from the duality between $\mathrm{Ind}_P^G \chi^{-1}$ and $M^{(\chi)}$. □

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